

## 关于孪生素数的一个注记

胡泽春

四川大学数学学院 四川成都

**【摘要】**在这篇注记中，我们对数列  $\{S_n\}$  进行一些讨论，其中  $S_n$  表示第  $n$  对孪生素数的和（我们把 (3,5) 作为第一对孪生素数）。我们将提出几个关于  $\{S_n\}$  的猜想，其中一个为孪生素数猜想的加强形式。

**【关键词】**孪生素数；孪生素数猜想

### A Note on Twin Prime Numbers

Zechun Hu

College of Mathematics, Sichuan University, Chengdu, Sichuan, China

**【Abstract】**In this note, we make some discussions about the number sequence  $\{S_n\}$ , where  $S_n$  is the sum of the  $n$  th pair of twin prime numbers (we regard (3,5) as the first pair). We will introduce several conjectures on the sequence  $\{S_n\}$ , one of which is a stronger version of twin prime number conjecture.

**【Keywords】**Twin Prime Number; Twin Prime Number Conjecture

#### 1 引言

根据算术基本定理（每个大于 1 的自然数都可以唯一地表示为素数的乘积），我们知道全体素数是构建整个自然数这个大厦的基石。哥德巴赫猜想告诉我们要构建大于 5 的偶数，我们不用素数相乘而只需要使用两个素数的和。再考虑到孪生素数的定义可知，孪生素数对的和似乎比较特殊。在这篇注记中，我们对数列  $\{S_n\}$  进行一些讨论，其中  $S_n$  表示第  $n$  对孪生素数的和（我们把 (3,5) 作为第一对孪生素数）， $n=1,2,\dots$ 。我们将提出几个关于  $\{S_n\}$  的猜想，其中一个为孪生素数猜想的加强形式。

关于哥德巴赫猜想，请参考 Chen [1]。关于孪生素数猜想，请参考 Zhang [2], Motohashi [3] 及文中的参考文献。

撰写这个注记的一个动机是来自我的感觉 --- 数列  $\{S_n\}$  是神秘的、有趣的。我希望这个注记能够引起部分数论学家对数列  $\{S_n\}$  进行深入的研究。这个注记于 2017 年 8 月 23 日完成，在网络平台 Researchgate 上进行了公开。

#### 2 孪生素数对的和及素因子分解

以下我们对前 394 对孪生素数求和，然后对和数进行素数乘积分解，黑体的素数表示在分解中首

次出现的素数（所有计算是利用计算器手工完成）。

$$S_1:=3+5=8=2^3$$

$$S_2:=5+7=12=2^2*3$$

$$S_3:=11+13=24=2^3*3$$

$$S_4:=17+19=36=2^2*3^2$$

$$S_5:=29+31=60=2^2*3*5$$

$$S_6:=41+43=84=2^2*3*7$$

$$S_7:=59+61=120=2^3*3*5$$

$$S_8:=71+73=144=2^4*3^2$$

$$S_9:=101+103=204=2^2*3*17$$

$$S_{10}:=107+109=216=2^3*3^3$$

$$S_{11}:=137+139=276=2^2*3*23$$

$$S_{12}:=149+151=300=2^2*3*5^2$$

$$S_{13}:=179+181=360=2^3*3^2*5$$

$$S_{14}:=191+193=384=2^7*3$$

$$S_{15}:=197+199=396=2^2*3^2*11$$

$$S_{16}:=227+229=456=2^4*3*7$$

$$S_{17}:=239+241=480=2^5*3*5$$

$$S_{18}:=269+271=540=2^2*3^3*5$$

$$S_{19}:=281+283=564=2^2*3*47$$

$$S_{20}:=311+313=624=2^4*3*13$$

$$S_{21}:=347+349=696=2^3*3*29$$

$$\begin{aligned}
S_{22} &:= 419 + 421 = 840 = 2^3 * 3 * 5 * 7 \\
S_{23} &:= 431 + 433 = 864 = 2^5 * 3^3 \\
S_{24} &:= 461 + 463 = 924 = 2^2 * 3 * 7 * 11 \\
S_{25} &:= 521 + 523 = 1044 = 2^2 * 3^2 * 29 \\
S_{26} &:= 569 + 571 = 1140 = 2^2 * 3^3 * 29 \\
S_{27} &:= 599 + 601 = 1200 = 2^4 * 3 * 5^2 \\
S_{28} &:= 617 + 619 = 1236 = 2^2 * 3 * \mathbf{103} \\
S_{29} &:= 641 + 643 = 1284 = 2^2 * 3 * \mathbf{107} \\
S_{30} &:= 659 + 661 = 1320 = 2^3 * 3 * 5 * 11 \\
S_{31} &:= 809 + 811 = 1620 = 2^2 * 3^4 * 5 \\
S_{32} &:= 821 + 823 = 1644 = 2^2 * 3 * \mathbf{137} \\
S_{33} &:= 827 + 829 = 1656 = 2^3 * 3^2 * 23 \\
S_{34} &:= 857 + 859 = 1716 = 2^2 * 3 * 11 * 13 \\
S_{35} &:= 881 + 883 = 1764 = 2^2 * 3^2 * 7^2 \\
S_{36} &:= 1019 + 1021 = 2040 = 2^3 * 3 * 5 * 17 \\
S_{37} &:= 1031 + 1033 = 2064 = 2^3 * 3^5 \\
S_{38} &:= 1049 + 1051 = 2100 = 2^2 * 3 * 5 * \mathbf{31} \\
S_{39} &:= 1061 + 1063 = 2124 = 2^2 * 3^3 * \mathbf{59} \\
S_{40} &:= 1091 + 1093 = 2184 = 2^3 * 3 * 7 * 13 \\
S_{41} &:= 1151 + 1153 = 2304 = 2^8 * 3^2 \\
S_{42} &:= 1229 + 1231 = 2460 = 2^2 * 3 * 5 * \mathbf{41} \\
S_{43} &:= 1277 + 1279 = 2556 = 2^2 * 3^2 * \mathbf{71} \\
S_{44} &:= 1289 + 1291 = 2580 = 2^2 * 3 * 5 * \mathbf{43} \\
S_{45} &:= 1301 + 1303 = 2604 = 2^2 * 3 * 7 * 31 \\
S_{46} &:= 1319 + 1321 = 2640 = 2^4 * 3 * 5 * 11 \\
S_{47} &:= 1427 + 1429 = 2856 = 2^3 * 3 * 7 * \mathbf{19} \\
S_{48} &:= 1451 + 1453 = 2904 = 2^3 * 3 * 11^2 \\
S_{49} &:= 1481 + 1483 = 2964 = 2^2 * 3 * 13 * 19 \\
S_{50} &:= 1487 + 1489 = 2976 = 2^5 * 3 * 31 \\
S_{51} &:= 1607 + 1609 = 3216 = 2^4 * 3 * \mathbf{67} \\
S_{52} &:= 1619 + 1621 = 3240 = 2^3 * 3^3 * 5 \\
S_{53} &:= 1667 + 1669 = 3336 = 2^3 * 3 * \mathbf{139} \\
S_{54} &:= 1697 + 1699 = 3396 = 2^2 * 3 * \mathbf{283} \\
S_{55} &:= 1721 + 1723 = 3444 = 2^2 * 3 * 7 * 41 \\
S_{56} &:= 1787 + 1789 = 3576 = 2^3 * 3 * \mathbf{149} \\
S_{57} &:= 1871 + 1873 = 3744 = 2^5 * 3^2 * 13 \\
S_{58} &:= 1877 + 1879 = 3756 = 2^2 * 3 * \mathbf{313} \\
S_{59} &:= 1931 + 1933 = 3864 = 2^3 * 3 * 7 * 23 \\
S_{60} &:= 1949 + 1951 = 3900 = 2^2 * 3 * 5^2 * 13 \\
S_{61} &:= 1997 + 1999 = 3996 = 2^2 * 3^3 * \mathbf{37} \\
S_{62} &:= 2027 + 2029 = 4056 = 2^3 * 3 * 13^2 \\
S_{63} &:= 2081 + 2083 = 4164 = 2^2 * 3 * \mathbf{347} \\
S_{64} &:= 2087 + 2089 = 4176 = 2^4 * 3^2 * 29 \\
S_{65} &:= 2111 + 2113 = 4224 = 2^7 * 3 * 11 \\
S_{66} &:= 2129 + 2131 = 4260 = 2^2 * 3 * 5 * 71 \\
S_{67} &:= 2141 + 2143 = 4284 = 2^2 * 3^2 * 7 * 17 \\
S_{68} &:= 2237 + 2239 = 4476 = 2^2 * 3 * \mathbf{373} \\
S_{69} &:= 2267 + 2269 = 4536 = 2^3 * 3^4 * 7 \\
S_{70} &:= 2309 + 2311 = 4620 = 2^2 * 3 * 5 * 7 * 11 \\
S_{71} &:= 2339 + 2341 = 4680 = 2^3 * 3^3 * 5 * 13 \\
S_{72} &:= 2381 + 2383 = 4764 = 2^2 * 3^4 * 11 \\
S_{73} &:= 2549 + 2551 = 5100 = 2^2 * 3 * 5^2 * 17 \\
S_{74} &:= 2591 + 2593 = 5184 = 2^6 * 3^4 \\
S_{75} &:= 2657 + 2659 = 5316 = 2^2 * 3 * \mathbf{443} \\
S_{76} &:= 2687 + 2689 = 5376 = 2^7 * 3 * 7 \\
S_{77} &:= 2711 + 2713 = 5424 = 2^4 * 3 * \mathbf{113} \\
S_{78} &:= 2729 + 2731 = 5460 = 2^2 * 3 * 5 * 7 * 13 \\
S_{79} &:= 2789 + 2791 = 5580 = 2^2 * 3^2 * 5 * 31 \\
S_{80} &:= 2801 + 2803 = 5604 = 2^2 * 3 * \mathbf{467} \\
S_{81} &:= 2969 + 2971 = 5940 = 2^2 * 3^3 * 5 * 11 \\
S_{82} &:= 2999 + 3001 = 6000 = 2^2 * 5^3 \\
S_{83} &:= 3119 + 3121 = 6240 = 2^5 * 3 * 5 * 13 \\
S_{84} &:= 3167 + 3169 = 6336 = 2^6 * 3^2 * 11 \\
S_{85} &:= 3251 + 3253 = 6504 = 2^3 * 3 * \mathbf{271} \\
S_{86} &:= 3257 + 3259 = 6516 = 2^2 * 3^2 * \mathbf{181} \\
S_{87} &:= 3299 + 3301 = 6600 = 2^3 * 3 * 5^2 * 11 \\
S_{88} &:= 3329 + 3331 = 6660 = 2^2 * 3 * 5 * 37 \\
S_{89} &:= 3359 + 3361 = 6720 = 2^6 * 3 * 5 * 7 \\
S_{90} &:= 3371 + 3373 = 6744 = 2^3 * 3 * \mathbf{281} \\
S_{91} &:= 3389 + 3391 = 6780 = 2^2 * 3 * 113 \\
S_{92} &:= 3461 + 3463 = 6924 = 2^2 * 3 * \mathbf{577} \\
S_{93} &:= 3467 + 3469 = 6936 = 2^3 * 3 * 17^2 \\
S_{94} &:= 3527 + 3529 = 7056 = 2^4 * 3^2 * 7^2 \\
S_{95} &:= 3539 + 3541 = 7080 = 2^3 * 3 * 5 * 59 \\
S_{96} &:= 3557 + 3559 = 7116 = 2^2 * 3 * \mathbf{593} \\
S_{97} &:= 3581 + 3583 = 7164 = 2^2 * 3^2 * \mathbf{199} \\
S_{98} &:= 3671 + 3673 = 7344 = 2^4 * 3^3 * 17 \\
S_{99} &:= 3767 + 3769 = 7536 = 2^4 * 3 * \mathbf{157} \\
S_{100} &:= 3821 + 3823 = 7644 = 2^2 * 3 * 7^2 * 13 \\
S_{101} &:= 3851 + 3853 = 7704 = 2^3 * 3^2 * 17 \\
S_{102} &:= 3917 + 3919 = 7836 = 2^2 * 3 * \mathbf{653} \\
S_{103} &:= 3929 + 3931 = 7860 = 2^2 * 3 * 5 * \mathbf{131}
\end{aligned}$$

S_104:=4001+4003=8004=2 <sup>2</sup> * 3* 23* 29	S_145:=6131+6133=12264=2 <sup>3</sup> * 3* 7* <b>73</b>
S_105:=4019+4021=8040=2 <sup>3</sup> * 3* 5* 67	S_146:=6197+6199=12396=2 <sup>2</sup> * 3* <b>1033</b>
S_106:=4049+4051=8100=2 <sup>2</sup> * 3 <sup>4</sup> * 5	S_147:=6269+6271=12540=2 <sup>2</sup> * 3* 5* 11* 19
S_107:=4091+4093=8184=2 <sup>3</sup> * 3* 11* 31	S_148:=6299+6301=12600=2 <sup>3</sup> * 3 <sup>2</sup> * 5 <sup>2</sup> * 7
S_108:=4127+4129=8256=2 <sup>6</sup> * 3* 43	S_149:=6359+6361=12720=2 <sup>4</sup> * 3* 5* <b>53</b>
S_109:=4157+4159=8316=2 <sup>2</sup> * 3 <sup>3</sup> * 7* 11	S_150:=6449+6451=12900=2 <sup>2</sup> * 3* 5 <sup>2</sup> * 43
S_110:=4217+4219=8436=2 <sup>2</sup> * 3* 19* 37	S_151:=6551+6553=13104=2 <sup>4</sup> * 3 <sup>2</sup> * 7* 13
S_111:=4229+4231=8460=2 <sup>2</sup> * 3 <sup>2</sup> * 5* 47	S_152:=6569+6571=13140=2 <sup>2</sup> * 3 <sup>2</sup> * 5* 73
S_112:=4241+4243=8484=2 <sup>2</sup> * 3* 7* <b>101</b>	S_153:=6659+6661=13320=2 <sup>3</sup> * 3 <sup>2</sup> * 5* 37
S_113:=4259+4261=8520=2 <sup>3</sup> * 3* 5* 71	S_154:=6689+6691=13380=2 <sup>2</sup> * 3* 5* <b>223</b>
S_114:=4271+4273=8544=2 <sup>5</sup> * 3* <b>89</b>	S_155:=6701+6703=13404=2 <sup>2</sup> * 3* <b>1117</b>
S_115:=4337+4339=8676=2 <sup>2</sup> * 3 <sup>2</sup> * <b>241</b>	S_156:=6761+6763=13524=2 <sup>2</sup> * 3* 7 <sup>2</sup> * 23
S_116:=4421+4423=8844=2 <sup>3</sup> * 3 <sup>2</sup> * 11* 67	S_157:=6779+6781=13560=2 <sup>3</sup> * 3* 5* 113
S_117:=4481+4483=8964=2 <sup>2</sup> * 3 <sup>3</sup> * <b>83</b>	S_158:=6791+6793=13584=2 <sup>4</sup> * 3* 283
S_118:=4517+4519=9036=2 <sup>2</sup> * 3 <sup>2</sup> * <b>251</b>	S_159:=6827+6829=13656=2 <sup>3</sup> * 3* <b>569</b>
S_119:=4547+4549=9096=2 <sup>3</sup> * 3* <b>379</b>	S_160:=6869+6871=13740=2 <sup>2</sup> * 3* 5* <b>229</b>
S_120:=4637+4639=9276=2 <sup>2</sup> * 3* <b>773</b>	S_161:=6947+6949=13896=2 <sup>3</sup> * 3 <sup>2</sup> * <b>193</b>
S_121:=4649+4651=9300=2 <sup>2</sup> * 3* 5 <sup>2</sup> * 31	S_162:=6959+6961=13920=2 <sup>5</sup> * 3* 5* 29
S_122:=4721+4723=9444=2 <sup>2</sup> * 3* <b>787</b>	S_163:=7127+7129=14256=2 <sup>4</sup> * 3 <sup>4</sup> * 11
S_123:=4787+4789=9576=2 <sup>3</sup> * 3 <sup>2</sup> * 7* 19	S_164:=7211+7213=14424=2 <sup>3</sup> * 3* <b>601</b>
S_124:=4799+4801=9600=2 <sup>7</sup> * 3* 5 <sup>2</sup>	S_165:=7307+7309=14616=2 <sup>3</sup> * 3 <sup>2</sup> * 7* 29
S_125:=4931+4933=9864=2 <sup>3</sup> * 3 <sup>2</sup> * 137	S_166:=7331+7333=14664=2 <sup>3</sup> * 3* 13* 47
S_126:=4967+4969=9936=2 <sup>4</sup> * 3 <sup>3</sup> * 23	S_167:=7349+7351=14700=2 <sup>2</sup> * 3* 5 <sup>2</sup> * 7 <sup>2</sup>
S_127:=5009+5011=10020=2 <sup>2</sup> * 3* 5* <b>167</b>	S_168:=7457+7459=14916=2 <sup>2</sup> * 3* 11* 113
S_128:=5021+5023=10044=2 <sup>2</sup> * 3 <sup>4</sup> * 31	S_169:=7487+7489=14976=2 <sup>7</sup> * 3* 39
S_129:=5099+5101=10200=2 <sup>3</sup> * 3* 5 <sup>2</sup> * 17	S_170:=7547+7549=15096=2 <sup>3</sup> * 3* 17* 37
S_130:=5231+5233=10464=2 <sup>5</sup> * 3* <b>109</b>	S_171:=7559+7561=15120=2 <sup>4</sup> * 3* 5* 7* 9
S_131:=5279+5281=10560=2 <sup>6</sup> * 3* 11	S_172:=7589+7591=15180=2 <sup>2</sup> * 3* 5* 11* 13
S_132:=5417+5419=10836=2 <sup>2</sup> * 3 <sup>2</sup> * 7* 43	S_173:=7757+7759=15516=2 <sup>2</sup> * 3 <sup>2</sup> * <b>431</b>
S_133:=5441+5443=10884=2 <sup>2</sup> * 3* <b>907</b>	S_174:=7877+7879=15756=2 <sup>2</sup> * 3* 13* 101
S_134:=5477+5479=10956=2 <sup>2</sup> * 3* 11* 83	S_175:=7949+7951=15900=2 <sup>2</sup> * 3* 5 <sup>2</sup> * 53
S_135:=5501+5503=11004=2 <sup>2</sup> * 3* 7* 131	S_176:=8009+8011=16020=2 <sup>2</sup> * 3 <sup>2</sup> * 5* 89
S_136:=5519+5521=11040=2 <sup>5</sup> * 3* 5* 23	S_177:=8087+8089=16176=2 <sup>4</sup> * 3* <b>337</b>
S_137:=5639+5641=11280=2 <sup>4</sup> * 3* 5* 7 <sup>2</sup>	S_178:=8219+8221=16440=2 <sup>3</sup> * 3* 5* 137
S_138:=5651+5653=11304=2 <sup>3</sup> * 3 <sup>2</sup> * 157	S_179:=8231+8233=16464=2 <sup>3</sup> * 3* 7 <sup>3</sup>
S_139:=5657+5659=11316=2 <sup>2</sup> * 3* 23* 41	S_180:=8291+8293=16584=2 <sup>3</sup> * 3* <b>691</b>
S_140:=5741+5743=11484=2 <sup>2</sup> * 3 <sup>2</sup> * 11* 29	S_181:=8387+8389=16776=2 <sup>3</sup> * 3 <sup>2</sup> * <b>233</b>
S_141:=5849+5851=11700=2 <sup>2</sup> * 3 <sup>2</sup> * 5* 13	S_182:=8429+8431=16860=2 <sup>2</sup> * 3* 5* 281
S_142:=5867+5869=11736=2 <sup>3</sup> * 3 <sup>2</sup> * <b>163</b>	S_183:=8537+8539=17076=2 <sup>2</sup> * 3* <b>1423</b>
S_143:=5879+5881=11760=2 <sup>4</sup> * 3* 5* 7 <sup>2</sup>	S_184:=8597+8599=17196=2 <sup>2</sup> * 3* <b>1433</b>
S_144:=6089+6091=12180=2 <sup>2</sup> * 3* 5* 7* 29	S_185:=8627+8629=17256=2 <sup>3</sup> * 3* <b>719</b>

S_186:=8819+8821=17640=2 <sup>3</sup> * 3 <sup>2</sup> * 5* 7 <sup>2</sup>	S_227:=11351+11353=22704=2 <sup>4</sup> * 3* 11* 43
S_187:=8837+8839=17676=2 <sup>2</sup> * 3 <sup>2</sup> * <b>491</b>	S_228:=11489+11491=22980=2 <sup>2</sup> * 3* 5* <b>383</b>
S_188:=8861+8863=17724=2 <sup>2</sup> * 3* 7* <b>211</b>	S_229:=11549+11551=23100=2 <sup>2</sup> * 3* 5 <sup>2</sup> * 73
S_189:=8969+8971=17940=2 <sup>2</sup> * 3* 5* 13* 23	S_230:=11699+11701=23400=2 <sup>3</sup> * 3 <sup>2</sup> * 5 <sup>2</sup> * 11
S_190:=8999+9001=18000=2 <sup>4</sup> * 3 <sup>2</sup> * 5 <sup>3</sup>	S_231:=11717+11719=23436=2 <sup>2</sup> * 3 <sup>3</sup> * 7* 31
S_191:=9011+9013=18024=2 <sup>3</sup> * 3* <b>751</b>	S_232:=11777+11779=23556=2 <sup>2</sup> * 3* 13* <b>151</b>
S_192:=9041+9043=18084=2 <sup>2</sup> * 3* 11* 137	S_233:=11831+11833=23664=2 <sup>4</sup> * 3* 17* 29
S_193:=9239+9241=18480=2 <sup>4</sup> * 3* 5* 7* 11	S_234:=11939+11941=23880=2 <sup>3</sup> * 3* 5* 199
S_194:=9281+9283=18564=2 <sup>2</sup> * 3 <sup>2</sup> * 7 <sup>2</sup> * 11	S_235:=11969+11971=23940=2 <sup>2</sup> * 3 <sup>2</sup> * 5* 7* 19
S_195:=9341+9343=18684=2 <sup>2</sup> * 3 <sup>3</sup> * <b>173</b>	S_236:=12041+12043=24084=2 <sup>2</sup> * 3 <sup>3</sup> * 223
S_196:=9419+9421=18840=2 <sup>3</sup> * 3* 5* 157	S_237:=12071+12073=24144=2 <sup>4</sup> * 3* <b>503</b>
S_197:=9431+9433=18864=2 <sup>4</sup> * 3 <sup>2</sup> * 131	S_238:=12107+12109=24216=2 <sup>3</sup> * 3* <b>1009</b>
S_198:=9437+9439=18876=2 <sup>2</sup> * 3* 11 <sup>2</sup> * 13	S_239:=12161+12163=24324=2 <sup>2</sup> * 3* <b>2027</b>
S_199:=9461+9463=18924=2 <sup>2</sup> * 3* 17* 83	S_240:=12239+12241=24480=2 <sup>5</sup> * 3 <sup>2</sup> * 5* 17
S_200:=9629+9631=19260=2 <sup>2</sup> * 3 <sup>2</sup> * 5* 107	S_241:=12251+12253=24504=2 <sup>3</sup> * 3* <b>1021</b>
S_201:=9677+9679=19356=2 <sup>2</sup> * 3* <b>1613</b>	S_242:=12377+12379=24756=2 <sup>2</sup> * 3* <b>2063</b>
S_202:=9719+9721=19440=2 <sup>4</sup> * 3 <sup>5</sup> * 5	S_243:=12539+12541=25080=2 <sup>3</sup> * 3* 5* 11* 19
S_203:=9767+9769=19536=2 <sup>4</sup> * 3* 11* 37	S_244:=12611+12613=25224=2 <sup>3</sup> * 3* <b>1051</b>
S_204:=9857+9859=19716=2 <sup>2</sup> * 3* 31* 53	S_245:=12821+12823=25644=2 <sup>2</sup> * 3* <b>2137</b>
S_205:=9929+9931=19860=2 <sup>2</sup> * 3* 5* <b>331</b>	S_246:=12917+12919=25836=2 <sup>2</sup> * 3* <b>2153</b>
S_206:=10007+10009=20016=2 <sup>4</sup> * 3 <sup>2</sup> * 139	S_247:=13001+13003=26004=2 <sup>2</sup> * 3* 11* <b>197</b>
S_207:=10037+10039=20076=2 <sup>2</sup> * 3* 7* <b>239</b>	S_248:=13007+13009=26016=2 <sup>5</sup> * 3* 271
S_208:=10067+10069=20136=2 <sup>3</sup> * 3* <b>839</b>	S_249:=13217+13219=26436=2 <sup>2</sup> * 3* <b>2203</b>
S_209:=10091+10093=20184=2 <sup>3</sup> * 3* 29 <sup>2</sup>	S_250:=13337+13339=26676=2 <sup>2</sup> * 3 <sup>3</sup> * 13* 19
S_210:=10139+10141=20280=2 <sup>3</sup> * 3* 5* 13 <sup>2</sup>	S_251:=13397+13399=26796=2 <sup>2</sup> * 3* 7* 11* 29
S_211:=10271+10273=20544=2 <sup>3</sup> * 3* 23* 37	S_252:=13679+13681=27360=2 <sup>5</sup> * 3 <sup>2</sup> * 5* 19
S_212:=10301+10303=20604=2 <sup>2</sup> * 3* 17* 101	S_253:=13691+13693=27384=2 <sup>3</sup> * 3* 7* 163
S_213:=10331+10333=20664=2 <sup>3</sup> * 3 <sup>2</sup> * 7* 41	S_254:=13709+13711=27420=2 <sup>2</sup> * 3* 5* <b>457</b>
S_214:=10427+10429=20856=2 <sup>3</sup> * 3* 11* <b>79</b>	S_255:=13721+13723=27444=2 <sup>2</sup> * 3* <b>2287</b>
S_215:=10457+10459=20916=2 <sup>2</sup> * 3 <sup>2</sup> * 7* 83	S_256:=13757+13759=27516=2 <sup>2</sup> * 3* <b>2293</b>
S_216:=10499+10501=21000=2 <sup>3</sup> * 3* 5 <sup>3</sup> * 7	S_257:=13829+13831=27660=2 <sup>2</sup> * 3* <b>461</b>
S_217:=10529+10531=21060=2 <sup>2</sup> * 3* 5 <sup>2</sup> * 71	S_258:=13877+13879=27756=2 <sup>2</sup> * 3 <sup>3</sup> * <b>257</b>
S_218:=10709+10711=21420=2 <sup>2</sup> * 3 <sup>2</sup> * 5* 7* 17	S_259:=13901+13903=27804=2 <sup>2</sup> * 3* 7* 331
S_219:=10859+10861=21720=2 <sup>3</sup> * 3* 5* 181	S_260:=13931+13933=27864=2 <sup>3</sup> * 3 <sup>4</sup> * 43
S_220:=10889+10891=21780=2 <sup>2</sup> * 3 <sup>2</sup> * 5* 11 <sup>2</sup>	S_261:=13997+13999=27996=2 <sup>2</sup> * 3* <b>2333</b>
S_221:=10937+10939=21876=2 <sup>2</sup> * 3* <b>1823</b>	S_262:=14009+14011=28020=2 <sup>2</sup> * 3* 467
S_222:=11057+11059=22116=2 <sup>2</sup> * 3* 19* <b>97</b>	S_263:=14081+14083=28164=2 <sup>2</sup> * 3* <b>2347</b>
S_223:=11069+11071=22140=2 <sup>2</sup> * 3 <sup>3</sup> * 5* 41	S_264:=14249+14251=28500=2 <sup>2</sup> * 3* 5 <sup>3</sup> * 19
S_224:=11117+11119=22236=2 <sup>2</sup> * 3* 17* 109	S_265:=14321+14323=28644=2 <sup>2</sup> * 3* 7* 11* 31
S_225:=11159+11161=22320=2 <sup>4</sup> * 3 <sup>2</sup> * 5* 31	S_266:=14387+14389=28776=2 <sup>3</sup> * 3* 11* 109
S_226:=11171+11173=22344=2 <sup>3</sup> * 3* 7 <sup>2</sup> * 19	S_267:=14447+14449=28896=2 <sup>5</sup> * 3* 7* 43

$S_{268}:=14549+14551=29100=2^2 \cdot 3 \cdot 5^2 \cdot 97$   
 $S_{269}:=14561+14563=29124=2^2 \cdot 3^2 \cdot 809$   
 $S_{270}:=14591+14593=29184=2^9 \cdot 3 \cdot 19$   
 $S_{271}:=14627+14629=29256=2^3 \cdot 3 \cdot 23 \cdot 53$   
 $S_{272}:=14867+14869=29736=2^3 \cdot 3^2 \cdot 7 \cdot 59$   
 $S_{273}:=15137+15139=30276=2^2 \cdot 3 \cdot 29^2$   
 $S_{274}:=15269+15271=30540=2^2 \cdot 3 \cdot 5 \cdot 509$   
 $S_{275}:=15287+15289=30576=2^4 \cdot 3 \cdot 7^2 \cdot 13$   
 $S_{276}:=15329+15331=30660=2^2 \cdot 3 \cdot 5 \cdot 7 \cdot 73$   
 $S_{277}:=15359+15361=30720=2^{\{11\}} \cdot 3 \cdot 5$   
 $S_{278}:=15581+15583=31164=2^2 \cdot 3 \cdot 7^2 \cdot 53$   
 $S_{279}:=15641+15643=31284=2^2 \cdot 3^2 \cdot 11 \cdot 79$   
 $S_{280}:=15647+15649=31296=2^2 \cdot 3 \cdot 163$   
 $S_{281}:=15731+15733=31464=2^3 \cdot 3^2 \cdot 19 \cdot 23$   
 $S_{282}:=15737+15739=31476=2^2 \cdot 3 \cdot 43 \cdot 61$   
 $S_{283}:=15887+15889=31776=2^5 \cdot 3 \cdot 331$   
 $S_{284}:=15971+15973=31944=2^3 \cdot 3 \cdot 11^3$   
 $S_{285}:=16061+16063=32124=2^2 \cdot 3 \cdot 2677$   
 $S_{286}:=16067+16069=32136=2^3 \cdot 3 \cdot 13 \cdot 103$   
 $S_{287}:=16139+16141=32280=2^3 \cdot 3 \cdot 5 \cdot 269$   
 $S_{288}:=16187+16189=32376=2^3 \cdot 3 \cdot 19 \cdot 71$   
 $S_{289}:=16229+16231=32460=2^2 \cdot 3 \cdot 5 \cdot 541$   
 $S_{290}:=16361+16363=32724=2^2 \cdot 3^4 \cdot 101$   
 $S_{291}:=16451+16453=32904=2^3 \cdot 3^2 \cdot 457$   
 $S_{292}:=16631+16633=33264=2^4 \cdot 3^2 \cdot 7 \cdot 11$   
 $S_{293}:=16649+16651=33300=2^2 \cdot 3 \cdot 5^2 \cdot 37$   
 $S_{294}:=16691+16693=33384=2^3 \cdot 3 \cdot 13 \cdot 107$   
 $S_{295}:=16829+16831=33660=2^2 \cdot 3^2 \cdot 5 \cdot 11^*$   
 $S_{296}:=16901+16903=33804=2^2 \cdot 3^2 \cdot 313$   
 $S_{297}:=16979+16981=33960=2^3 \cdot 3 \cdot 5 \cdot 283$   
 $S_{298}:=17027+17029=34056=2^3 \cdot 3^2 \cdot 11 \cdot 43$   
 $S_{299}:=17189+17191=34380=2^2 \cdot 3^2 \cdot 5 \cdot 191$   
 $S_{300}:=17207+17209=34416=2^4 \cdot 3 \cdot 239$   
 $S_{301}:=17291+17293=34584=2^3 \cdot 3 \cdot 11 \cdot 131$   
 $S_{302}:=17387+17389=34776=2^3 \cdot 3^3 \cdot 7 \cdot 23$   
 $S_{303}:=17417+17419=34836=2^2 \cdot 3 \cdot 2903$   
 $S_{304}:=17489+17491=34980=2^2 \cdot 3 \cdot 5 \cdot 11 \cdot 53$   
 $S_{305}:=17579+17581=35160=2^3 \cdot 3 \cdot 5 \cdot 293$   
 $S_{306}:=17597+17599=35196=2^2 \cdot 3 \cdot 7 \cdot 419$   
 $S_{307}:=17657+17659=35316=2^2 \cdot 3^4 \cdot 109$

$S_{308}:=17681+17683=35364=2^2 \cdot 3 \cdot 7 \cdot 421$   
 $S_{309}:=17747+17749=35496=2^3 \cdot 3^2 \cdot 17 \cdot 29$   
 $S_{310}:=17789+17791=35580=2^2 \cdot 3 \cdot 5 \cdot 593$   
 $S_{311}:=17837+17839=35676=2^2 \cdot 3^2 \cdot 991$   
 $S_{312}:=17909+17911=35820=2^2 \cdot 3^2 \cdot 5 \cdot 199$   
 $S_{313}:=17921+17923=35844=2^2 \cdot 3 \cdot 29 \cdot 103$   
 $S_{314}:=17957+17959=35916=2^2 \cdot 3 \cdot 41 \cdot 73$   
 $S_{315}:=17987+17989=35976=2^3 \cdot 3 \cdot 1499$   
 $S_{316}:=18041+18043=36084=2^2 \cdot 3 \cdot 31 \cdot 97$   
 $S_{317}:=18047+18049=36096=2^8 \cdot 3 \cdot 47$   
 $S_{318}:=18059+18061=36120=2^3 \cdot 3 \cdot 5 \cdot 7 \cdot 43$   
 $S_{319}:=18119+18121=36240=2^4 \cdot 3 \cdot 5 \cdot 151$   
 $S_{320}:=18131+18133=36264=2^3 \cdot 3 \cdot 1511$   
 $S_{321}:=18251+18253=36504=2^3 \cdot 3^3 \cdot 13^2$   
 $S_{322}:=18287+18289=36576=2^5 \cdot 3^2 \cdot 127$   
 $S_{323}:=18311+18313=36624=2^4 \cdot 3 \cdot 7 \cdot 109$   
 $S_{324}:=18521+18523=37044=2^2 \cdot 3^3 \cdot 7^3$   
 $S_{325}:=18539+18541=37080=2^3 \cdot 3^2 \cdot 5 \cdot 103$   
 $S_{326}:=18911+18913=37824=2^6 \cdot 3 \cdot 197$   
 $S_{327}:=18917+18919=37836=2^2 \cdot 3^2 \cdot 1051$   
 $S_{328}:=19079+19081=38160=2^4 \cdot 3 \cdot 5 \cdot 53$   
 $S_{329}:=19139+19141=38280=2^3 \cdot 3 \cdot 5 \cdot 11 \cdot 29$   
 $S_{330}:=19181+19183=38364=2^2 \cdot 3 \cdot 23 \cdot 139$   
 $S_{331}:=19211+19213=38424=2^3 \cdot 3 \cdot 1601$   
 $S_{332}:=19379+19381=38760=2^2 \cdot 3 \cdot 17 \cdot 19^*$   
 $S_{333}:=19421+19423=38844=2^2 \cdot 3 \cdot 13 \cdot 83$   
 $S_{334}:=19427+19429=38856=2^3 \cdot 3 \cdot 1619$   
 $S_{335}:=19469+19471=38940=2^2 \cdot 3 \cdot 5 \cdot 11 \cdot 59$   
 $S_{336}:=19541+19543=39084=2^2 \cdot 3 \cdot 3257$   
 $S_{337}:=19697+19699=39396=2^2 \cdot 3 \cdot 7^2 \cdot 67$   
 $S_{338}:=19751+19753=39504=2^4 \cdot 3 \cdot 823$   
 $S_{339}:=19841+19843=39684=2^2 \cdot 3 \cdot 3307$   
 $S_{340}:=19889+19891=39780=2^2 \cdot 3^2 \cdot 5 \cdot 13^*$   
 $S_{341}:=19961+19963=39924=2^2 \cdot 3^2 \cdot 1109$   
 $S_{342}:=19991+19993=39984=2^4 \cdot 3 \cdot 7^2 \cdot 17$   
 $S_{343}:=20021+20023=40044=2^2 \cdot 3 \cdot 47 \cdot 71$   
 $S_{344}:=20147+20149=40296=2^3 \cdot 3 \cdot 23 \cdot 73$   
 $S_{345}:=20231+20233=40464=2^4 \cdot 3^2 \cdot 281$   
 $S_{346}:=20357+20359=40716=2^2 \cdot 3^3 \cdot 13 \cdot 29$

17

23

17

$S_{347}:=20441+20443=40884=2^2 \cdot 3 \cdot 3407$   
 $S_{348}:=20477+20479=40956=2^2 \cdot 3 \cdot 3413$   
 $S_{349}:=20507+20509=41016=2^3 \cdot 3 \cdot 1709$   
 $S_{350}:=20549+20551=41100=2^4 \cdot 3 \cdot 5^2 \cdot 137$   
 $S_{351}:=20639+20641=41280=2^6 \cdot 3 \cdot 5 \cdot 43$   
 $S_{352}:=20717+20719=41436=2^2 \cdot 3^2 \cdot 1151$   
 $S_{353}:=20747+20749=41496=2^3 \cdot 3 \cdot 7 \cdot 13 \cdot 19$   
 $S_{354}:=20771+20773=41544=2^3 \cdot 3^2 \cdot 577$   
 $S_{355}:=20807+20809=41616=2^4 \cdot 3^2 \cdot 17^2$   
 $S_{356}:=20897+20899=41796=2^2 \cdot 3^5 \cdot 43$   
 $S_{357}:=20981+20983=41964=2^2 \cdot 3 \cdot 13 \cdot 269$   
 $S_{358}:=21011+21013=42024=2^3 \cdot 3 \cdot 17 \cdot 103$   
 $S_{359}:=21017+21019=42036=2^2 \cdot 3 \cdot 31 \cdot 113$   
 $S_{360}:=21059+21061=42120=2^3 \cdot 3^4 \cdot 5 \cdot 13$   
 $S_{361}:=21191+21193=42384=2^4 \cdot 3 \cdot 883$   
 $S_{362}:=21317+21319=42636=2^2 \cdot 3 \cdot 7 \cdot 17 \cdot 19$   
 $S_{363}:=21377+21379=42756=2^2 \cdot 3 \cdot 7 \cdot 509$   
 $S_{364}:=21491+21493=42984=2^3 \cdot 3^3 \cdot 199$   
 $S_{365}:=21521+21523=43044=2^2 \cdot 3 \cdot 17 \cdot 211$   
 $S_{366}:=21557+21559=43116=2^2 \cdot 3 \cdot 3593$   
 $S_{367}:=21587+21589=43176=2^2 \cdot 3 \cdot 7 \cdot 257$   
 $S_{368}:=21599+21601=43200=2^6 \cdot 3^3 \cdot 5^2$   
 $S_{369}:=21611+21613=43224=2^3 \cdot 3 \cdot 1801$   
 $S_{370}:=21647+21649=43296=2^5 \cdot 3 \cdot 11 \cdot 41$   
 $S_{371}:=21737+21739=43476=2^2 \cdot 3 \cdot 3623$   
 $S_{372}:=21839+21841=43680=2^5 \cdot 3 \cdot 5 \cdot 7 \cdot 13$   
 $S_{373}:=22037+22039=44076=2^2 \cdot 3 \cdot 3673$   
 $S_{374}:=22091+22093=44184=2^3 \cdot 3 \cdot 7 \cdot 263$   
 $S_{375}:=22109+22111=44220=2^2 \cdot 3 \cdot 5 \cdot 7 \cdot 67$   
 $S_{376}:=22157+22159=44316=2^2 \cdot 3^2 \cdot 1231$   
 $S_{377}:=22271+22273=44544=2^3 \cdot 3^2 \cdot 617$   
 $S_{378}:=22277+22279=44556=2^2 \cdot 3 \cdot 47 \cdot 79$   
 $S_{379}:=22367+22369=44736=2^6 \cdot 3 \cdot 233$   
 $S_{380}:=22481+22483=44964=2^2 \cdot 3^2 \cdot 1249$   
 $S_{381}:=22541+22543=45084=2^2 \cdot 3 \cdot 17^3$   
 $S_{382}:=22571+22573=45144=2^3 \cdot 3^3 \cdot 11 \cdot 19$   
 $S_{383}:=22619+22621=45240=2^3 \cdot 3 \cdot 5 \cdot 13 \cdot 29$   
 $S_{384}:=22637+22639=45276=2^2 \cdot 3 \cdot 7^3 \cdot 11$   
 $S_{385}:=22697+22699=45396=2^2 \cdot 3^2 \cdot 13 \cdot 97$   
 $S_{386}:=22739+22741=45480=2^3 \cdot 3 \cdot 5 \cdot 379$   
 $S_{387}:=22859+22861=45720=2^3 \cdot 3^2 \cdot 5 \cdot 127$

$S_{388}:=22961+22963=45924=2^2 \cdot 3 \cdot 43 \cdot 89$   
 $S_{389}:=23027+23029=46056=2^3 \cdot 3 \cdot 19 \cdot 101$   
 $S_{390}:=23039+23041=46080=2^4 \cdot 10 \cdot 3^2 \cdot 5$   
 $S_{391}:=23057+23059=46116=2^2 \cdot 3^3 \cdot 7 \cdot 61$   
 $S_{392}:=23201+23203=46404=2^2 \cdot 3^2 \cdot 1289$   
 $S_{393}:=23291+23293=46584=2^3 \cdot 3^2 \cdot 647$   
 $S_{394}:=23369+23371=46740=2^2 \cdot 3 \cdot 5 \cdot 19 \cdot 41$

2.1 猜想 1

对于素数  $p$ , 定义

$$T(p) := \inf\{k \geq 1: p | S_k\}.$$

对于前 40 个素数, 我们有

$T(2)=1$	$T(3)=2$	$T(5)=5$	$T(7)=6$
$T(11)=15$	$T(13)=20$	$T(17)=9$	$T(19)=4$
$T(23)=11$	$T(29)=21$	$T(31)=38$	$T(37)=61$
$T(41)=42$	$T(43)=44$	$T(47)=19$	$T(53)=149$
$T(59)=39$	$T(61)=282$	$T(67)=51$	$T(71)=43$
$T(73)=145$	$T(79)=214$	$T(83)=117$	$T(89)=114$
$T(97)=222$	$T(101)=112$	$T(103)=28$	$T(107)=29$
$T(109)=130$	$T(113)=77$	$T(127)=322$	$T(131)=103$
$T(137)=32$	$T(139)=53$	$T(149)=56$	$T(151)=232$
$T(157)=99$	$T(163)=142$	$T(167)=127$	$T(173)=195$

基于以上的信息和我们的直觉, 我们引入如下的孪生素数猜想的加强形式:

**猜想 1:** 数列  $\{S_n, n=1, 2, 3, \dots\}$  包含所有的素数作为它们的素因子。

**注记:** (1) 很显然, 上述猜想要成立必须要有无穷多对孪生素数, 也就是说孪生素数猜想必须成立。

(2) 我曾让我以前指导过的硕士生崔振(后在英国取得博士学位, 目前在山东师范大学工作) 编程进行了验证, 前 500 个素数都找到了相应的孪生素数对。

2.2 猜想 2

对于  $k=1, 2, 3, \dots$ , 定义

$N(k) := \#\{p: p \text{ 为素数, 存在 } j \in \{1, \dots, k\} \text{ 使得 } p | S_j\},$   
 即  $N(k)$  表示前  $k$  对孪生素数对的和所包含的不同素因子的个数。我们有

$$\begin{aligned}
 N(1) &= 1, \\
 N(2) &= N(3) = N(4) = 2, \\
 N(5) &= 3 \\
 N(6) &= N(7) = N(8) = 4
 \end{aligned}$$

$N(9)=N(10)=5$	$N(120)=N(121)=46$
$N(11)=N(12)=N(13)=N(14)=6$	$N(122)=\cdots=N(126)=47$
$N(15)=N(16)=N(17)=N(18)=7$	$N(127)=N(128)=N(129)=48$
$N(19)=8$	$N(130)=N(131)=N(132)=49$
$N(20)=9$	$N(133)=\cdots=N(141)=50$
$N(21)=\cdots=N(27)=10$	$N(142)=N(143)=N(144)=51$
$N(28)=11$	$N(145)=52$
$N(29)=N(30)=N(31)=12$	$N(146)=N(147)=N(148)=53$
$N(32)=\cdots=N(38)=13$	$N(149)=\cdots=N(153)=54$
$N(38)=14$	$N(154)=55$
$N(39)=N(40)=N(41)=15$	$N(155)=\cdots=N(158)=56$
$N(42)=16$	$N(159)=57$
$N(43)=17$	$N(160)=58$
$N(44)=N(45)=N(46)=18$	$N(161)=N(162)=N(163)=59$
$N(47)=N(48)=N(49)=N(50)=19$	$N(164)=\cdots=N(172)=60$
$N(51)=N(52)=20$	$N(173)=\cdots=N(176)=61$
$N(53)=21$	$N(177)=N(178)=N(179)=62$
$N(54)=N(55)=22$	$N(180)=63$
$N(56)=N(57)=23$	$N(181)=N(182)=64$
$N(58)=N(59)=N(60)=24$	$N(183)=65$
$N(61)=N(62)=25$	$N(184)=66$
$N(63)=N(67)=26$	$N(185)=N(186)=67$
$N(68)=N(69)=\cdots=N(74)=27$	$N(187)=68$
$N(75)=N(76)=28$	$N(188)=N(189)=N(190)=69$
$N(77)=N(78)=N(79)=29$	$N(191)=\cdots=N(194)=70$
$N(80)=N(81)=\cdots=N(84)=30$	$N(195)=\cdots=N(200)=71$
$N(85)=31$	$N(201)=\cdots=N(204)=72$
$N(86)=N(87)=N(88)=N(89)=32$	$N(205)=N(206)=73$
$N(90)=N(91)=33$	$N(207)=74$
$N(92)=\cdots=N(95)=34$	$N(208)=\cdots=N(213)=75$
$N(96)=35$	$N(214)=\cdots=N(220)=76$
$N(97)=N(98)=36$	$N(221)=77$
$N(99)=N(100)=N(101)=37$	$N(222)=\cdots=N(227)=78$
$N(102)=38$	$N(228)=\cdots=N(231)=79$
$N(103)=\cdots=N(111)=39$	$N(232)=\cdots=N(237)=80$
$N(112)=N(113)=40$	$N(238)=81$
$N(114)=41$	$N(239)=N(240)=82$
$N(115)=N(116)=42$	$N(241)=83$
$N(117)=43$	$N(242)=N(243)=84$
$N(118)=44$	$N(244)=85$
$N(119)=45$	$N(245)=86$

$N(246)=87$   
 $N(247)=N(248)=88$   
 $N(249)=\dots=N(253)=89$   
 $N(254)=90$   
 $N(255)=91$   
 $N(256)=92$   
 $N(257)=93$   
 $N(258)=N(259)=N(260)=94$   
 $N(261)=N(262)=95$   
 $N(263)=\dots=N(268)=96$   
 $N(169)=\dots=N(273)=97$   
 $N(274)=\dots=N(281)=98$   
 $N(282)=N(283)=N(284)=99$   
 $N(285)=N(286)=100$   
 $N(287)=N(288)=101$   
 $N(289)=\dots=N(298)=102$   
 $N(299)=\dots=N(302)=103$   
 $N(303)=N(304)=104$   
 $N(305)=105$   
 $N(306)=N(307)=106$   
 $N(308)=N(309)=N(310)=107$   
 $N(311)=\dots=N(314)=108$   
 $N(315)=\dots=N(319)=109$   
 $N(320)=N(321)=110$   
 $N(322)=\dots=N(330)=111$   
 $N(331)=N(332)=N(333)=112$   
 $N(334)=N(335)=113$   
 $N(336)=N(337)=114$   
 $N(338)=115$   
 $N(339)=N(340)=116$   
 $N(341)=\dots=N(346)=117$   
 $N(347)=118$   
 $N(348)=119$   
 $N(349)=N(350)=N(351)=120$   
 $N(352)=\dots=N(360)=121$   
 $N(361)=\dots=N(365)=122$   
 $N(366)=N(367)=N(368)=123$   
 $\dots\dots$

猜想 2: 存在一个正常数  $c$  使得对任意的  $k = 1, 2, 3, \dots, N(k) \geq ck$ .

联想到素数定理, 感觉上述猜想的结论太强, 提出以下较弱的版本 (2021 年 11 月 27 日提出):

猜想 2\*: 存在一个正常数  $c$  使得对任意的  $k = 1, 2, 3, \dots, N(k) \geq ck / \ln k$ .

3 数列  $\{S_{n+1} - S_n, n \geq 1\}$

根据第 2 节的信息, 我们有

$$S_2 - S_1 = 12 - 8 = 4$$

$$S_3 - S_2 = 24 - 12 = 12 = 12$$

$$S_4 - S_3 = 36 - 24 = 12 = 12$$

$$S_5 - S_4 = 60 - 36 = 24 = 12 * 2$$

$$S_6 - S_5 = 84 - 60 = 24 = 12 * 2$$

$$S_7 - S_6 = 120 - 84 = 36 = 12 * 3$$

$$S_8 - S_7 = 144 - 120 = 24 = 12 * 2$$

$$S_9 - S_8 = 204 - 144 = 60 = 12 * 5$$

$$S_{10} - S_9 = 216 - 204 = 12 = 12$$

$$S_{11} - S_{10} = 276 - 216 = 60 = 12 * 5$$

$$S_{12} - S_{11} = 300 - 276 = 24 = 12 * 2$$

$$S_{13} - S_{12} = 360 - 300 = 60 = 12 * 5$$

$$S_{14} - S_{13} = 384 - 360 = 24 = 12 * 2$$

$$S_{15} - S_{14} = 396 - 384 = 12 = 12$$

$$S_{16} - S_{15} = 456 - 396 = 60 = 12 * 5$$

$$S_{17} - S_{16} = 480 - 456 = 24 = 12 * 2$$

$$S_{18} - S_{17} = 540 - 480 = 60 = 12 * 5$$

$$S_{19} - S_{18} = 564 - 540 = 24 = 12 * 2$$

$$S_{20} - S_{19} = 624 - 564 = 60 = 12 * 5$$

$$S_{21} - S_{20} = 696 - 624 = 72 = 12 * 6$$

$$S_{22} - S_{21} = 840 - 696 = 144 = 12 * 12$$

$$S_{23} - S_{22} = 864 - 840 = 24 = 12 * 2$$

$$S_{24} - S_{23} = 924 - 864 = 60 = 12 * 5$$

$$S_{25} - S_{24} = 1044 - 924 = 120 = 12 * 10$$

$$S_{26} - S_{25} = 1140 - 1044 = 96 = 12 * 8$$

$$S_{27} - S_{26} = 1200 - 1140 = 60 = 12 * 5$$

$$S_{28} - S_{27} = 1236 - 1200 = 36 = 12 * 3$$

$$S_{29} - S_{27} = 1284 - 1236 = 48 = 12 * 4$$

$$S_{30} - S_{29} = 1320 - 1284 = 36 = 12 * 3$$

$$S_{31} - S_{30} = 1620 - 1320 = 300 = 12 * 25$$

$$S_{32} - S_{31} = 1644 - 1620 = 24 = 12 * 2$$

$$S_{33} - S_{32} = 1656 - 1644 = 12 = 12$$

$$S_{34} - S_{33} = 1716 - 1656 = 60 = 12 * 5$$

$$S_{35} - S_{34} = 1764 - 1716 = 48 = 12 * 4$$

$$S_{36} - S_{35} = 2040 - 1764 = 276 = 12 * 23$$

$$S_{37} - S_{36} = 2064 - 2040 = 24 = 12 * 2$$

$$S_{38} - S_{37} = 2100 - 2064 = 36 = 12 * 3$$

$$S_{39} - S_{38} = 2124 - 2100 = 24 = 12 * 2$$



$S_{40}-S_{39}=2184-2124=60=12*5$   
 $S_{41}-S_{40}=2304-2184=120=12*10$   
 $S_{42}-S_{41}=2460-2304=156=12*13$   
 $S_{43}-S_{42}=2556-2460=96=12*8$   
 $S_{44}-S_{43}=2580-2556=24=12*2$   
 $S_{45}-S_{44}=2604-2580=24=12*2$   
 $S_{46}-S_{45}=2640-2604=36=12*3$   
 $S_{47}-S_{46}=2856-2640=216=12*18$   
 $S_{48}-S_{47}=2904-2856=48=12*4$   
 $S_{49}-S_{48}=2964-2904=60=12*5$   
 **$S_{50}-S_{49}=2976-2964=12=12$**   
 $S_{51}-S_{50}=3216-2976=240=12*20$   
 $S_{52}-S_{51}=3240-3216=24=12*2$   
 $S_{53}-S_{52}=3336-3240=96=12*8$   
 $S_{54}-S_{53}=3396-3336=60=12*5$   
 $S_{55}-S_{54}=3444-3396=48=12*4$   
 $S_{56}-S_{55}=3576-3444=132=12*11$   
 $S_{57}-S_{56}=3744-3576=168=12*14$   
 **$S_{58}-S_{57}=3756-3744=12=12$**   
 $S_{59}-S_{58}=3864-3756=108=12*9$   
 $S_{60}-S_{59}=3900-3864=36=12*3$   
 $S_{61}-S_{60}=3996-3900=96=12*8$   
 $S_{62}-S_{61}=4056-3996=60=12*5$   
 $S_{63}-S_{62}=4164-4056=108=12*9$   
 **$S_{64}-S_{63}=4176-4164=12=12$**   
 $S_{65}-S_{64}=4224-4176=48=12*4$   
 $S_{66}-S_{65}=4260-4224=36=12*3$   
 $S_{67}-S_{66}=4284-4260=24=12*2$   
 $S_{68}-S_{67}=4476-4284=192=12*16$   
 $S_{69}-S_{68}=4536-4476=60=12*5$   
 $S_{70}-S_{69}=4620-4536=84=12*7$   
 $S_{71}-S_{70}=4680-4620=60=12*5$   
 $S_{72}-S_{71}=4764-4680=84=12*7$   
 $S_{73}-S_{72}=5100-4764=336=12*28$   
 $S_{74}-S_{73}=5184-5100=84=12*7$   
 $S_{75}-S_{74}=5316-5184=132=12*11$   
 $S_{76}-S_{75}=5376-5316=60=12*5$   
 $S_{77}-S_{76}=5424-5376=48=12*4$   
 $S_{78}-S_{77}=5460-5424=36=12*3$   
 $S_{79}-S_{78}=5580-5460=120=12*10$   
 $S_{80}-S_{79}=5604-5580=24=12*2$

$S_{81}-S_{80}=5940-5604=336=12*28$   
 $S_{82}-S_{81}=6000-5940=60=12*5$   
 $S_{83}-S_{82}=6240-6000=240=12*20$   
 $S_{84}-S_{83}=6336-6240=96=12*8$   
 $S_{85}-S_{84}=6504-6336=168=12*14$   
 **$S_{86}-S_{85}=6516-6504=12=12$**   
 $S_{87}-S_{86}=6600-6516=84=12*7$   
 $S_{88}-S_{87}=6660-6600=60=12*5$   
 $S_{89}-S_{88}=6720-6660=60=12*5$   
 $S_{90}-S_{89}=6744-6720=24=12*2$   
 $S_{91}-S_{90}=6780-6744=36=12*3$   
 $S_{92}-S_{91}=6924-6780=144=12*12$   
 **$S_{93}-S_{92}=6936-6924=12=12$**   
 ... ..  
 **$S_{139}-S_{138}=11316-11304=12=12$**   
 ... ..  
 **$S_{198}-S_{197}=18876-18864=12=12$**   
 ... ..  
 **$S_{248}-S_{247}=26016-26004=12=12$**   
 ... ..  
 **$S_{139}-S_{138}=11316-11304=12=12$**   
 ... ..  
 **$S_{280}-S_{279}=31296-31284=12=12$**   
 ... ..  
 **$S_{282}-S_{281}=31476-31464=12=12$**   
 ... ..  
 **$S_{286}-S_{285}=32136-32124=12=12$**   
 ... ..  
 **$S_{317}-S_{316}=36096-36084=12=12$**   
 ... ..  
 **$S_{327}-S_{326}=37836-37824=12=12$**   
 ... ..  
 **$S_{334}-S_{333}=38856-38844=12=12$**   
 ... ..  
 **$S_{359}-S_{358}=42036-42024=12=12$**   
 ... ..  
 **$S_{378}-S_{377}=44556-44544=12=12$**

**猜想 3:** 存在无穷多个  $k$  使得  
 $S_{\{k+1\}}-S_k=12$ , 换句话说, 存在无穷多组“四胞胎素数”。

我相信这个猜想已经由别人提出过。

## 参考文献

- [1] J. R. Chen, On the representation of a larger even integer as the sum of a prime and the product of at most two primes. *Sci. Sinica*, 16 (1973), 157–176 .
- [2] Y. Zhang, Bounded gaps between primes. *Ann. of Math. (2)*, 179 (2014), no. 3, 1121–1174.
- [3] Y. Motohashi, The twin prime conjecture, arXiv: 1401.6614v2, 2014.

**收稿日期:** 2020 年 11 月 08 日

**出刊日期:** 2021 年 12 月 14 日

**引用本文:** 胡泽春, 关于孪生素数的一个注记[J]. *国际应用数学进展*, 2021, 3 : 42-51.

DOI: 10.12208/j.aam.20210003

**检索信息:** RCCSE 权威核心学术期刊数据库、中国知网 (CNKI Scholar)、万方数据 (WANFANG DATA)、Google Scholar 等数据库收录期刊

**版权声明:** ©2021 作者与开放获取期刊研究中心 (OAJRC) 所有。本文章按照知识共享署名许可条款发表。 <http://creativecommons.org/licenses/by/4.0/>



**OPEN ACCESS**